Identification of the Inverse Dynamics of Robot Manipulators with the Structured Kernel

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***Abstract*—The inverse dynamics model of robots is often the key for accurate control. Especially in the computed torque control, the nonlinearity and the friction can be compensated, leading to better performance. The inverse models, however, is not trivial. The traditional Euler-Lagrange model based on the rigid body assumption often underfits in the presence of frictions and requires tedious derivations; the learning-based model needs larger training data set, since the structure of the dynamics is not considered. To overcome the aforementioned issues, we propose a structured kernel to replace the rigid body model and combine it with the universal radial basis kernel by direct sum. The proposed structured kernel asymptotically has the same convergence rate as the traditional model, and is general regardless of the configuration of the robot. Therefore, no analytic derivation is needed. Together with the universal radial basis kernel, the proposed approach enjoys the advantages of both the conventional and the learning-based models. To verify the proposed method, the simulations are used to investigate the performance in terms of the prediction errors.**

I. Introduction

It is well known that a control loop that includes the inverse dynamics as the feed-forward compensation achieves better performance in tracking and force control [1]. In particular, the impedance control of robots requires the inverse dynamics model to perform the desired dynamics [2], or the prediction of human intention in the exoskeleton control [3]. Despite the appealing success of the inverse dynamics model, the application is limited in general due to the modeling difficulties. Traditionally, the dynamics model of the robots is derived based on Newton-Euler method or Euler-Lagrange formulation [4], which we refer them as the *rigid body model* hereafter. Assuming that each link of the robot is a rigid body and the friction can be neglected, the unknowns in the dynamics model consists of the *kinematic parameters* and the *dynamics parameters*, in which the kinematic parameters are referred to those specifying the traditional Denavit-Hartenberg model, whereas the dynamics parameters consists of the inertia matrix, and the position and the mass of the center of mass for each link. Traditionally, the calibration of the dynamics model can be categorized into two approaches according to the formulation. In the Newton-Euler method, the kinematic parameters is first calibrated using laser [5] or camera [6], and the dynamics parameters are identified joint by joint iteratively [7]. This approach restricts the calibration to be carried only in offline, and the most importantly, the uncertainty of the former links passes to the following links. An alternative solution is based on the Euler-Lagrange formulation [7]. This approach explores all the unknown terms in the dynamics model in terms of the nonlinear bases, and therefore the identification problem becomes an ordinary linear regression problem, which is then solved by least-squares. While the Euler-Lagrange based approach is popular especially in the adaptive control community [8], the number of expanded terms of the dynamics model grows, however, in the worst case exponentially with the degrees of freedom (DOF) of the robot. As a result, the computational burden may become intractable, which limits the usage for complex systems, e.g. robot manipulators with arbitrary Denavit-Hartenberg parameters. Also, none of the models above consider directly the frictions.

Considering the uncertainties due to frictions, joint flexibility, and manufacturing errors, the machine learning based inverse dynamics models haves been proposed [9-16]. The learning of the dynamics can be long dated back to the neural networks, and the following machines based on the reproducing kernel Hilbert space (RKHS), e.g. support vector machine and Gaussian process regression, etc. Despite the universality of these methods, the curse of dimensionality and the ability of generalization to the unseen data remain as the major issues. First, since the complexity of the underlying problem grows exponentially with the DOF of the robot, how to choose a proper kernel space and the regularization is the key to prevent the overfitting. Second, along with the curse of the dimensionality, the definition of sufficient rich data for a machine learning model in the learning the dynamics may vary and depend on the applications.

The popular radial basis kernel and those kernels that decay as the distance metric between the seen and the unseen data increases often fails to generalize well in learning the inverse dynamics. More specifically, they often underestimate the predicted torque. Since theses kernels are essentially based on the interpolation of the acquired training data, the sufficiently rich training data should cover all the frequencies and the magnitudes, or should be at least sufficiently rich in the space of possible reference trajectories so that the identically independently distributed (i.i.d.) sampling assumption holds. On the other hand, the analytic rigid body model can generalize well, if the effect of the unmodeled dynamics is limited and negligible. Since it captures the polynomial tendency of the generalized force with respect to the accelerations and the velocities, the sufficiently rich data

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do not require the trajectories of all frequencies and magnitudes. That is, the model can predict for the high-speed trajectory with large magnitude well even with limited training data.

Recently, Duy et al.[13] showed that the prediction and the control is more accurate by incorporating both the rigid body model and the radial basis kernel compared to the model used only either of them alone, in which the rigid body model is derived based on the Euler-Lagrange method. Along with this trend, we too propose to use the fusion the two models here. Compared to [13], no explicit derivation of the rigid body model is needed here. By formulating the proper RKHS, we can directly model the rigid body dynamics without referring to the Euler-Lagrange method, which is often tedious even with the symbolic mathematics toolbox. Also, we introduce an additional parameter to control the complexity of the overall model, so the model can regularize the learning in the presence of the curse of the dimensionality as the DOF increase. Since the proposed RKHS is radically the rigid body dynamics, it converges to the analytic model uniformly.

In summary, we introduce the new structured reproducing kernel and further, in a multiple kernel fashion, incorporate it with the universal radial basis kernel, which is used to model the nonlinear dynamics and the frictions. To verify the proposed kernel, we test the ability in the simulations. For the rest of this paper, it is organized as follows. Section II shows the main result of this work, the explicit formulation of the kernel space, and Section III shows how the multiple kernels is formulated to learn the inverse dynamics model with the standard kernel methods, such as support vector regression and kernel ridge regression. In Section IV, the simulation results are presented, which is then discussed in Section V. Finally, we give a short conclusion in Section VI.

II. Reproducing Kernel Hilbert Space of Rigid Body Dynamics

By identifying the structure of the function space with Euler-Lagrange Method, the corresponding RKHS of the inverse dynamics can be derived. The objective, here, is to identify the smallest possible RKHS in the sense of dimensionality that includes the function space of the inverse dynamics. In this paper, we assume the robot is holonomic and serial with all rotary joints, but this framework can be easily extended to the cases with prismatic joints accordingly. We omitted here for the compactness. Also, we mention that the models of all the joints are learned independently.

A. The Euler-Lagrange Method

We begin with analyzing the Euler-Lagrange formulation of the dynamics of the robot [7]. For a *N*-DOF robot, let $q \in \mathbb{R}^N$ be the generalized coordinates, i.e. the joint angle for a rotary robot, and let

$$T(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} \tag{1}$$

$$= \frac{1}{2}\dot{q}^T \sum_{i=1}^{N}\Big[m_i J_{v_i}(q)^T J_{v_i}(q) + J_{\omega_i}(q)^T R_i(q)\Omega_i R_i(q)^T J_{\omega_i}(q)\Big]\dot{q}$$

$$U(q) = \sum_{i=1}^{N} -m_i g^T r_{ci}(q) \tag{2}$$

be the kinematic energy and the potential energy respectively, where m_i is the mass, r_{ci} is the position of the center of the mass, Ω_i is the inertia tensor matrix, J_{v_i} is the Jacobian matrix of linear velocity, J_{ω_i} is the Jacobian matrix of angular velocity, R_i is the rotational matrix between the inertial frame to joint frame of link i, and g is the gravitational acceleration vector, and $M(q) \in \mathbb{R}^{N\times N}$ is the generalized inertia matrix of the whole robot defined as in (1). With the kinematic energy and the potential energy, we define the Lagrangian as

$$L := T - U\ . \tag{3}$$

The Euler-Lagrange equation shows the generalized force is actually the image of the Lagrangian under a linear map, i.e.

$$\left(\frac{d}{dt}\frac{\partial}{\partial \dot{q}_n} - \frac{\partial}{\partial q_n}\right)L = \tau_n\ , \tag{4}$$

where q_n is the *n*th generalized coordinate, and τ_n is the *n*th generalized force. Or we can write it more compactly in the matrix form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau\ , \tag{5}$$

where $C(q,\dot{q}) \in \mathbb{R}^{N\times N}$ is the Coriolis/centrifugal matrix, $G(q) \in \mathbb{R}^N$ is the gravitational term, $\tau \in \mathbb{R}^N$ is the vector of generalized forces.

In the context of robotics, (5) are referred to the dynamics of Euler-Lagrange method, in contrast to the iterative Newton-Euler method. It can be shown that the unknowns in (5) including both the kinematic and dynamic parameters can be arranged in a linear regressor form, so they can be identified by the ordinary linear regression offline, or by the canonical adaptive law online. However, the number of the bases and the unknowns in the worst case grow exponentially with *N*, if there is no cancellation due to the zero terms in the kinematic parameters, e.g. zero link length, link offset in the Denavit-Hartenberg parameters. Also, the derivation of the exact formulation of (5) for general robots is actually intricate and computationally intractable even for the symbolic mathematics toolbox.

B. Reproducing Kernel Hilbert Space of Rigid Body Dynamics

A RKSH $\mathcal{H}$ [17] is vector space of continuous functions with the reproducing property

$$f(x) = \langle f, k_x \rangle_{\mathcal{H}}\ ,\ \forall f \in \mathcal{H}\ , \tag{6}$$

where k_x is the corresponding reproducing kernel of $\mathcal{H}$, so the evaluation of a function $f \in \mathcal{H}$ is the projection on the kernel k_x. As a result, the learning problem of the unknown function f, i.e. the system identification, given some finite observations becomes the inference problem given the projection of f on some subspace in $\mathcal{H}$ defined by the observation [18], which can be solved the standard kernel methods e.g. regulated least-square or support vector machine. However, the choice of RKHS actually affects the learning greatly. A RKHS with small effective dimensionality or size in terms of k_x can learn and converge faster. Therefore, the chosen RKHS should capture the structure of the unknown function f.

The objective is to find a particular RKHS that is large enough to contain (4) and yet small enough to prevent overfitting given finite observations. Also the RKHS should be endowed with computationally efficient reproducing kernel. We denote $\mathcal{H}_L$ the RKHS that contains (4) with such characteristics. Now we shall derive the analytic form of the reproducing kernel of $\mathcal{H}_L$. Let $\mathcal{X}$ be the set of all possible states $(q,\dot{q},\ddot{q})$ of the robot, and with the abuse of notation we may write, for example, $q \in \mathcal{X}$ for simplicity. Since $\mathcal{X}$ is a bounded and closed subset in the Euclidean space, and (4) is a bounded and continuous functional on $\mathcal{X}$, there exists at least one RKHS containing (4). First, we define the following RKHSs,

$$\mathcal{H}_{\ddot{q}} = \overline{span}\{\frac{1}{\sqrt{N}}\ddot{q} \mid \ddot{q} \in \mathcal{X}\} \tag{7}$$

$$\mathcal{H}_{\dot{q}\otimes\dot{q}} = \overline{span}\{\frac{1}{\sqrt{N}}\dot{q} \otimes \frac{1}{\sqrt{N}}\dot{q} \mid \dot{q} \in \mathcal{X}\} \tag{8}$$

$$\mathcal{H}_{\chi} = \overline{span}\{\chi(q) \mid q \in \mathcal{X}\}, \tag{9}$$

where the nonlinear map

$$\begin{aligned} &\chi : \mathcal{X} \to \mathbb{R}^{3^N} \\ &x = (q,\dot{q},\ddot{q}) \to \underset{n\in\mathbb{N}_N}{\otimes} (\cos q_n, \sin q_n, 1) \end{aligned}, \tag{10}$$

the bar denotes the completion of a metric space, $\mathbb{N}_N := \{1,...,N\}$, and $\sqrt{N}$ is introduced as a normalization factor. Namely, $\mathcal{H}_{\ddot{q}}$ contains the linear functions of $\ddot{q} \in \mathcal{X}$, and $\mathcal{H}_{\dot{q}\otimes\dot{q}}$ contains the quadratic functions of $\dot{q} \in \mathcal{X}$, and so on.

Proposition

The kinematic energy and the potential energy of the rigid body dynamics lie in the following RKHS,

$$T \in \mathcal{H}_{\dot{q}\otimes\dot{q}} \otimes \mathcal{H}_{\chi} \otimes \mathcal{H}_{\chi} =: \mathcal{H}_T, \tag{11}$$

$$U \in \mathcal{H}_{\chi} =: \mathcal{H}_U; \tag{12}$$

Proof:

Since the Jacobian matrices can be shown to be

$$J_{v_i} = [\frac{\partial r_{ci}}{\partial q_1} \quad ... \quad \frac{\partial r_{ci}}{\partial q_N}], \tag{13}$$

$$J_{\omega_i} = [\rho_1 z_0 \quad ... \quad \rho_N z_{N-1}], \tag{14}$$

where

$$\rho_n = \begin{cases} 0, & \text{if joint } n \text{ prismatic} \\ 1, & \text{if joint } n \text{ rotary} \end{cases} \tag{15}$$

z_{n-1} is the axis of the nth generalized coordinate, and $n \in \mathbb{N}_N$. We note that the order of the subscript follows the traditional Denavit-Hartenberg convention, in which the frame i-1 is defined with respect to link i, and the two endpoints of link i are joint i and joint i+1. Although we assume all the joints are rotary, the derivation for prismatic joints is simpler and can be done in the same fashion. To claim Proposition, it is sufficient to show J_{v_i}, $R_i(q)^T J_{\omega_i}(q) \in \mathcal{H}_\chi$, where it is obvious that $J_{v_i} \in \mathcal{H}_\chi$ since $r_{ci} \in \mathcal{H}_\chi$ and the linear operator $\partial / \partial q_n$ maps all the elements in $\mathcal{H}_\chi$ to the subspace in $\mathcal{H}_\chi$. For the angular velocity, we have

$$R_i(q)^T \rho_n z_{n-1} = \rho_n R_{i-1}^0(q)^T R_n^0(q) e_3 = \rho_n R_n^{i-1}(q) e_3, \tag{16}$$

where $e_3 \in \mathbb{R}^3$ is the standard basis, and therefore it is in $\mathcal{H}_\chi$. As for the potential energy, the derivation is similar.

Q.E.D.

After identifying the RKHSs of the kinematic energy and the potential energy, the RKHS $\mathcal{H}_L$ should contains the image of $\mathcal{H}_T \oplus \mathcal{H}_U$ under the linear map (4).

Theorem

Let $\mathrm{Im}(\mathcal{H}_T \oplus \mathcal{H}_U)_n$ *be the image of* $\mathcal{H}_T \oplus \mathcal{H}_U$ *under the linear map,*

$$T_n := (\frac{d}{dt}\frac{\partial}{\partial \dot{q}_n} - \frac{\partial}{\partial q_n}), \ n \in \mathbb{N}_N. \tag{17}$$

Then $\mathrm{Im}(\mathcal{H}_T \oplus \mathcal{H}_U)_n$ *can be included in the following RKHS*

$$\mathcal{H}_L := (\mathcal{H}_{\ddot{q}} \oplus \mathcal{H}_{\dot{q}\otimes\dot{q}}) \otimes (\mathcal{H}_\chi \otimes \mathcal{H}_\chi) \oplus \mathcal{H}_\chi \setminus \mathcal{H}_1, \tag{18}$$

for all $n \in \mathbb{N}_N$, *where* $\mathcal{H}_1$ *is the space of constant function.*

The RKHS $\mathcal{H}_L$ is indeed computationally efficient and can model the inverse dynamics with the reproducing kernel,

$$\begin{aligned} \hat{k}_{\mathcal{H}_L}(x,y) = &(\langle \ddot{q}_y, \ddot{q}_x \rangle + \langle \dot{q}_y, \dot{q}_x \rangle^2) \prod_{i\in\mathbb{N}_N} (\cos(q_{x,i} - q_{y,i}) + 1)^2 \\ &+ \prod_{i\in\mathbb{N}_N} (\cos(q_{x,i} - q_{y,i}) + 1) - 1 \end{aligned} \tag{19}$$

We omitted the proof here for the compactness.

In fact, it is only slightly larger than the smallest RKHS possible by observing that the smallest RKHS to contain τ_n for all $n \in \mathbb{N}_N$ is of size N times larger than $\mathrm{Im}(\mathcal{H}_T \oplus \mathcal{H}_U)_n$. However, such a RKHS may loss the easily manipulated reproducing kernel function, whereas in $\mathcal{H}_L$ the reproducing kernel has a simple closed form.

To this end, if (19) is used unreservedly, the curse of dimensionality may occur. We can observe that $\dim(\mathcal{H}_L) = \mathcal{O}(6^N N^2)$, and therefore the limited size of training data can never catch up the size of the hypothesis space. This obstruction can be circumvented by introducing an additional regularization parameter σ, and by scaling $\mathcal{H}_L$, we can obtain a new reproducing kernel

$$k_{\mathcal{H}_L}(x,y) = (\langle \ddot{q}_y, \ddot{q}_x \rangle + \langle \dot{q}_y, \dot{q}_x \rangle^2) k_\chi(q_x, q_y)^2 + k_\chi(q_x, q_y) - 1 \tag{20}$$

where

$$k_\chi(q_x, q_y) := \prod_{i \in \mathbb{N}_N} \frac{(\cos(q_{x,i} - q_{y,i}) + \sigma)}{1+\sigma} - (\frac{\sigma}{1+\sigma})^N + 1 \tag{21}$$

In this scaled $\mathcal{H}_L$, the contributions of high-order terms are penalized, though the overall dimensionality is the same. The additional parameter σ, selected by cross-validation, can guide the machine to bias to the correct parameterization. One can observe that the high-order terms in (4) are those without the cancellation due to the DH parameters, which shows the parameterization in (20) is a good prior knowledge to learn the inverse dynamics. Finally, we remark that $\mathcal{H}_L$ is essentially the same as the rigid body dynamics, but no derivation of nonlinear bases in (5) is needed anymore, and the computational time decreases due to the computational efficient kernel (20).

III. Learning the Inverse Dynamics in Multiple-Kernel Formulation

By learning the inverse dynamics, we mean learning the mapping from the states of the dynamics and the actuation force τ_a, that is $\Gamma : (q, \dot{q}, \ddot{q}) \to \tau_a$ such that

$$(\frac{d}{dt}\frac{\partial}{\partial \dot{q}_n} - \frac{\partial}{\partial q_n})L = \tau_n = \tau_{a,n} + \tau_{f,n} \tag{22}$$

holds for all $n \in \mathbb{N}_N$, where τ_f is the force due to frictions and other unmodeled dynamics, and the subscript denotes the *n*th component. In the presence of τ_f, the inverse map Γ is not well defined in general, whereas the inverse map from $(q, \dot{q}, \ddot{q})$ to τ_n always exists. Let μ_x be the probability measure on $\mathcal{X}$, the learning of inverse dynamics is aimed to find the actuation force $\hat{\tau}_a$ such that,

$$\hat{\tau}_{a,n} = \arg\min_{\tilde{\tau}_{a,n} \in \mathcal{H}} \left\| \tilde{\tau}_{a,n} + \tau_{f,n} - \tau_n \right\|^2_{\mathcal{L}^2(\mathcal{X}, \mu_x)} \tag{23}$$

for all $n \in \mathbb{N}_N$, where $\mathcal{H}$ is the hypothesis space.

As mentioned earlier and evidenced in [13], the combination of the rigid body dynamics and the radial basis kernel can model the dynamics better, since the rigid body dynamics captures quickly the structured component while the radial basis kernel can approximate universally any function. Therefore, to model the inverse dynamics we combine the proposed kernel (20) with a general radial basis kernel *rbfs*

$$k_{rbfs}(x,y) := \exp(-\frac{\|\cos q_x - \cos q_y\|^2 + \|\sin q_x - \sin q_y\|^2 + \|\dot{q}_x - \dot{q}_y\|^2 + \|\ddot{q}_x - \ddot{q}_y\|^2}{2\sigma}) \tag{24}$$

where the trigonometric mappings are introduced to better model the effect of the rotary joints, in contrast to the traditionally used radial basis kernel *rbf* in modeling (5)

$$k_{rbf}(x,y) := \exp(-\frac{\|q_x - q_y\|^2 + \|\dot{q}_x - \dot{q}_y\|^2 + \|\ddot{q}_x - \ddot{q}_y\|^2}{2\sigma}) \tag{25}$$

In terms of (23), it is equivalent to choose

$$\mathcal{H} = \mathcal{H}_L \oplus \mathcal{H}_{rbfs}, \tag{26}$$

where $\mathcal{H}_{rbfs}$ is the RKHS with reproducing kernel (24). Since $\tau_n \in \mathcal{H}_L$ and $\mathcal{H}_{rbfs}$ is universal, we can have

$$\begin{aligned} &\min_{\tilde{\tau}_{a,n} \in \mathcal{H}} \left\| \tilde{\tau}_{a,n} + \tau_{f,n} - \tau_n \right\|^2_{\mathcal{L}^2(\mathcal{X}, \mu_x)} \\ &= \min_{\tilde{\tau}_{f,n} \in \mathcal{H}_{rbfs}} \left\| \tilde{\tau}_{f,n} - \tau_{f,n} \right\|^2_{\mathcal{L}^2(\mathcal{X}, \mu_x)} = Var_x\{\tau_{f,n}\} \end{aligned} \tag{27}$$

That is, the modeling error is due to the variance of τ_f with respect to the probability measure μ_x as the number of observations goes to infinity.

In implementation, the inverse dynamics is learned by the combination of kernels (20) and (24),

$$k_{mkl} = \delta k_{\mathcal{H}_L} + (1-\delta) k_{rbfs}, \tag{28}$$

where $\delta \in [0,1]$ chosen be cross-validation in the experiments and $k_{\mathcal{H}_L}$ and k_{rbfs} are normalized so that the traces of the empirical kernel matrices are the same, and solved by the kernel ridge regression or the support vector regression.

IV. Simulations

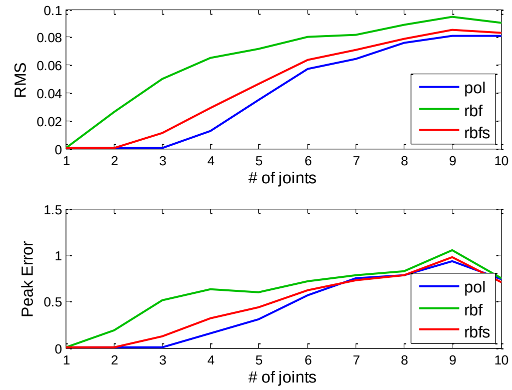
In this section, the simulation results are presented. We want to compare the generalization of the proposed kernels (20) and (28), and the traditional learning-based approaches (24) and (25). In each of the following simulations, we show the testing error with respect to the complexity of the underlying model, i.e. the DOF of the robot, in different scenarios: with or without the presence of measurement noise and nonlinear frictions. In the following, for each of DOF, five different robots with random kinematic and dynamic parameters are used as the plant to be learned. We remark that the parameters are sampled from a bounded uniform distribution so that all the parameters are physically feasible, e.g. the inertia matrix are always positive definite. For each of the robot, 5000 training data and another 5000 testing data with angular positions, angular velocities, and angular accelerations sampled from the uniform distribution are used for the validation. Given the uniformly random states, the kinematic and the dynamic parameters, the generalized forces in (4) are computed using Newton-Euler method iteratively. Therefore, the data of the ideal robot dynamics can be obtained. As for the unmodeled dynamics, the adopted noise is the zero-mean Gaussian noise, the viscous friction is modeled by the force linear to the generalized velocity, and the Coulomb friction is modeled as the sign function of the generalized velocity. To learn the unknown model, we use the least-square regularized learning, i.e. the kernel ridge regression, and the kernel parameters and the parameter that controls the tradeoff between the complexity of the space and the fitting error all chosen by the 3-fold cross-validation. The optimal parameters is chosen to be the combination of parameters that minimize the empirically expected prediction error, and the whole training data set is used to retrain the final model with the optimal parameter. Finally, to verify the result, the performances are shown in terms of prediction errors overall all the generalized coordinates both in root mean square (RMS) and the peak error (PE). Also, we note that all the generalized force in the simulation are normalized within [-1,1] for comparison. In the following, the proposed kernel (20) is denoted by *pol*, (25) is denoted by *rbf*, (24) is denoted by *rbfs*, and (28) is denoted by *mkl*.

Fig. 1 shows the simulation results of the ideal robot dynamics without any friction and noises. Fig. 1 (a) and (b) shows the prediction error of the normalized generalized force in terms of RMS and PE with respect to the complexity of the model, respectively. We remark here that the dimensionality of the proposed kernel is actually analytic

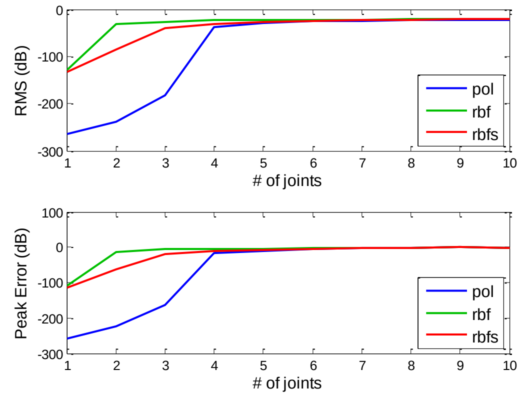
$$\dim(\mathcal{H}_L) = (N + \frac{1}{2}N(N+1))6^N + 3^N - 1, \qquad (29)$$

which is the upper bound of possible terms in a general Euler-Lagrange model. Since 5000 training data is sufficient to cover the whole space in terms of the dimensionality for a robot with $N \le 3$, we can see a clear boundary on $N = 3$ in *pol*. On the other hand, 5000 training data no longer covers the whole hypothesis space for robot with $N > 3$. In this situation, the performance of the kernel depends on the quality of the regularized parameters, i.e. σ in each kernel function. As mentioned in (29), the size of the training data may never catch up the dimensionality of the underlying model, so the performance of different models become close especially when $\ell << N^2 6^N$. Also, we can observe *rbfs* outperforms the traditional *rbf*, since the characteristics of the rotary joints are better captured. Comparing all the models, the proposed *pol* consistently shows better performance, which is expected since it is radically the Euler-Lagrange model.

Fig. 2 shows the simulation results of the ideal robot dynamics with both frictions and noises. The Coulomb frictions and the viscous frictions are modeled as mentioned previously with the magnitude chosen randomly, and the noise is the zero-mean Gaussian noise with standard deviation 0.05. Compared to Fig. 1, the kernel *pol* performs badly in the presence of frictions. The performance can be increased, however, by introducing *rbfs* forming the kernel *mkl*. Since *mkl* captures partly the structure of the dynamics, the performance is consistently better than *rbf* and *rbfs*. Another feature is that all the models seem to learn similarly as N increases. This is because the effect of the frictions is neglectable when the training data is too scarce compared to the size of the space. In particular, *pol* learns as without friction and is better than both *rbf* and *rbfs*. The *mkl* model, on



(a)



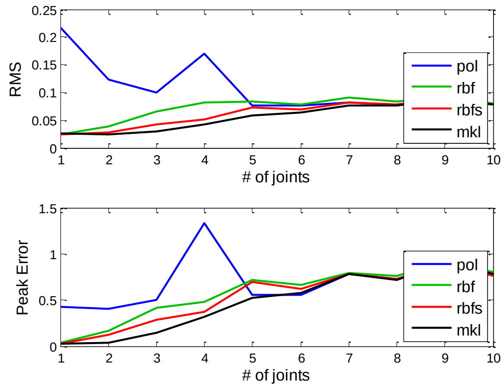
(b)

Fig. 1. Prediction error of the ideal model without measurement noise. (a) prediction error (b) prediction error shown in dB

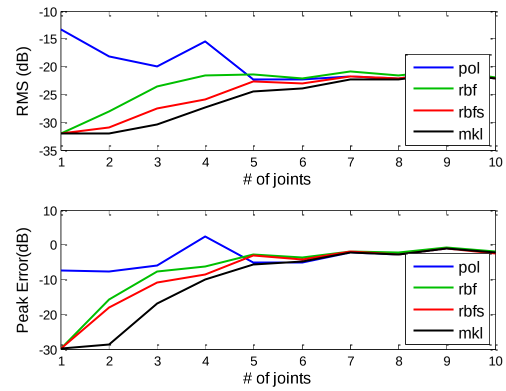
the contrary, performs better than the others regardless of the condition of frictions and noises. Finally, we note that the computational time of evaluating 1 testing samples given the model spanned by 5000 training samples in a standard PC with CPU i5-750 is bounded below 5ms. Therefore, the real-time computation is possible.

V. Discussions

In the simulation of ideal robot dynamics, the proposed *pol* kernel shows better generalization compared to the general learning kernel, since it captures the structure of the dynamics of the robot as the Euler-Lagrange model. However, *pol* may give unsatisfactory results when the friction cannot be neglected. To learn the unmodeled dynamics as well, we propose *mkl* to combine *pol* with *rbfs* by direct sum, where the weighting between two kernels δ is chosen by cross-validation instead of using the general multiple-kernel learning that automatically tunes the weighting for the following two reasons. First, the multiple-kernel learning tends to overfit when quality of the training data is poor or the size of the training data is small, since the multiple-kernel learning is actually an expectation maximization routine. Second, the precise value of unknown parameter in our experiments tends not to be decisive, so only a small set of parameters are needed to be tested in the cross-validation, which is much faster than the multiple kernel learning . The results show that *mkl* predicts better compared to the general



(a)



(b)

Fig. 2. Prediction error of the ideal model with measurement noises and frictions. (a) prediction error (b) prediction error shown in dB

kernels *rbf*, *rbfs* and the structured kernel *pol* used alone. In comparison with [13], the proposed *mkl* is expected to give a similar result, since regularized-least square is essentially the Gaussian regression without the predicting variance. However, one may expect that *mkl* may be worse than the fusion kernel in [13], since they use the explicit Euler-Lagrange model compared to the proposed *pol*. This is the necessary tradeoff between the size of the hypothesis space and the generality. Since *pol* is general to all the rigid body dynamics, *pol* uses larger hypothesis space than the Euler-Lagrange model given specific kinematic parameters. On the contrary, *pol* does not need any derivation, and can calibrate the kinematic parameters as well. Thus, we regard *mkl* as an efficient alternative, since they shared the same asymptotical learning rate.

VI. Conclusion

In this paper, we demonstrate how to design a reproducing kernel that naturally models the function space of the robot dynamics. By modeling the structure of the robot dynamics into the RKHS, the generalization of the proposed is better compared to the kernels used extensively in learning inverse dynamics in the literatures, which is evidenced both in *pol* and *rbfs* in the simulations. For the future works, the experiments of modeling the inverse dynamics and the associating feedforward compensations will be performed. Also, we would like to modify our framework so the information of the nominal plant can be included.

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